

CIRCULAR MOTION

Basic term Related to Angular Motion

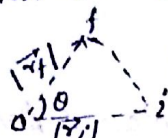
111- Angular displacement (θ) → Body Rotate w.r.t. fixed point. Angle b/w Initial & Final position vector.

- * When magnitude of Initial & Final position vector is same.
- * unit → Radian (r), Degree (°)
- * Dimensionless.

* Small Angular displacement is a Axial vector & Its direction is define from Right hand thumb rule.

* Large Angular displacement is scalar quantity.

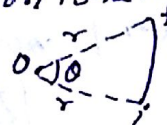
NOTE → AXIAL vector → Vector which is along the Rotational axis.



$$|\vec{r}_i| = |\vec{r}_f|$$

* ⇒ Angular displacement in one Rotation is 2π .

* ⇒ 'N' Rotation ⇒ $2\pi N = \theta$



$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$

$$\theta = \frac{\text{Arc}}{r}$$

$$\text{Arc} = \theta r$$

121- Angular velocity (ω) → Rate of change in Angular disp. represent Angular velocity.

* It is also Axial vector & direction // to the angular disp.

$$\vec{\omega} = \frac{\Delta \theta}{\Delta t} \quad * \text{unit} \rightarrow \text{R/sec} \quad * \vec{\omega} \parallel \vec{\theta}$$

$$1 \text{ Rotation} \Rightarrow \omega = \frac{\Delta \theta}{\Delta t} = \frac{2\pi}{T} = 2\pi n$$

Angular freq. or velocity

Time period

Frequency [Hz, c.p.s, p.m, R.p.m]

Types of Angular velocity

111- uniform Angular velocity
Direction + magnitude same.

111- Non-uniform Angular velocity
Direction change / magnitude change / Both change.

111- Inst. Angular velocity

$$\omega_t = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

1st derivative of Angular disp.

111- Avg. Angular velocity

$$\vec{\omega}_{\text{avg}} = \frac{\text{Total Angular disp.}}{\text{Total time}}$$

$$\vec{\omega}_{\text{avg}} = \frac{\vec{\theta}_1 + \vec{\theta}_2 + \vec{\theta}_3 + \dots + \vec{\theta}_N}{t_1 + t_2 + t_3 + \dots + t_N}$$

Standard Result

111- Same time Interval.

$$\omega_{\text{avg}} = \frac{\omega_1 + \omega_2 + \omega_3 + \dots + \omega_N}{N}$$

111- Same Angular displacement.

$$(\theta_1 = \theta_2 = \dots = \theta_N)$$

$$\omega_{\text{avg}} = \frac{1}{\frac{1}{\omega_1} + \frac{1}{\omega_2} + \frac{1}{\omega_3} + \dots + \frac{1}{\omega_N}}$$

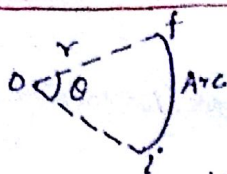
111- $\theta = f(\text{time}) \Rightarrow \text{function of time} \Rightarrow \omega = \frac{d\theta}{dt} = f(t)$

$$\langle \omega \rangle = \omega_{\text{avg}} = \frac{\int_{t_1}^{t_2} \omega(t) dt}{t_2 - t_1}$$

111- $\omega = f(\theta)$

$$\langle \omega \rangle = \frac{\int_{\theta_1}^{\theta_2} \omega(\theta) d\theta}{\theta_2 - \theta_1}$$

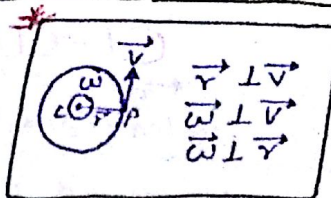
Relation b/w Linear velocity & Angular velocity



$$\theta = \frac{\text{Arc}}{r} \Rightarrow \text{Arc} = \theta r$$

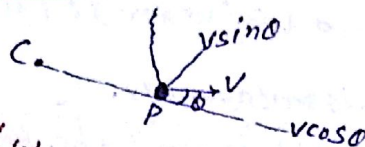
$$V = r\omega \quad \vec{V} = \vec{\omega} \times \vec{r}$$

Linear velocity Angular velocity Tangential vector Radial vector Angular vector



|V| -> Relative Angular velocity (ω_R)

$$\omega_R = \frac{V_T}{r} = \frac{V \sin \theta}{r}$$

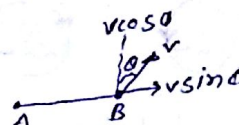


Relative Angular velocity at point 'P' w.r.t point 'C'

$$\omega_R = \frac{V \sin \theta}{r} = \frac{V \sin \theta}{r}$$

Relative Angular velocity at point 'B' w.r.t point 'A'

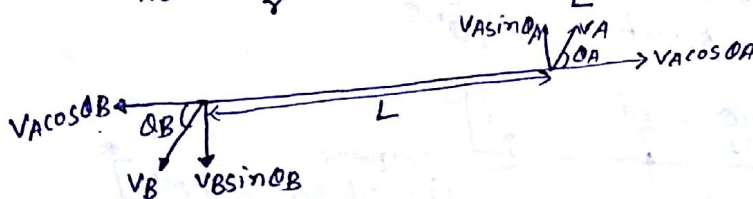
$$\omega_{BA} = \frac{V_L}{r} = \frac{V \cos \theta}{r}$$



#

Relative Angular velo of 'A' w.r.t 'B'

$$\omega_{AB} = \frac{(V_L)_{AB}}{r} = \frac{V_A \sin \theta_A - V_B \sin \theta_B}{L}$$

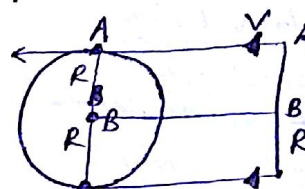


Particle rotate in a circular path of radius 'R' with velocity 'V'. then Relative Angular velocity of particle 'A' point 'A' w.r.t point 'B' & 'C'.

$$\omega_{AB} = \frac{V_L}{r} = \frac{V}{R}$$

$$\frac{\omega_{AB}}{\omega_{AC}} = \frac{V/R}{V/2R} = 2:1$$

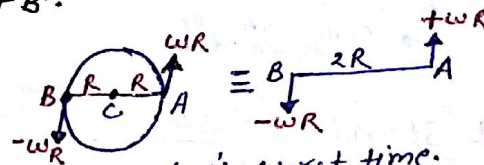
$$\omega_{AC} = \frac{V_L}{r} = \frac{V}{2R}$$



Two particle rotate in circular path of radius 'R' with angular velocity 'omega' at point 'A' & 'B'. Relative angular velocity of 'A' w.r.t 'B'.

$$\omega_{AB} = \frac{(V_L)_{AB}}{r} = \frac{\omega R + \omega R}{2R}$$

$$\omega_{AB} = \omega$$



|3| -> Angular Acceleration (α) -> Rate of change in Angular velocity w.r.t time.

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

* unit -> R/sec^2

* Angular Accn is axial vector & it is || to the angular velocity & Angular disp.

$$\vec{\alpha} \parallel \vec{\omega} \parallel \vec{\theta}$$

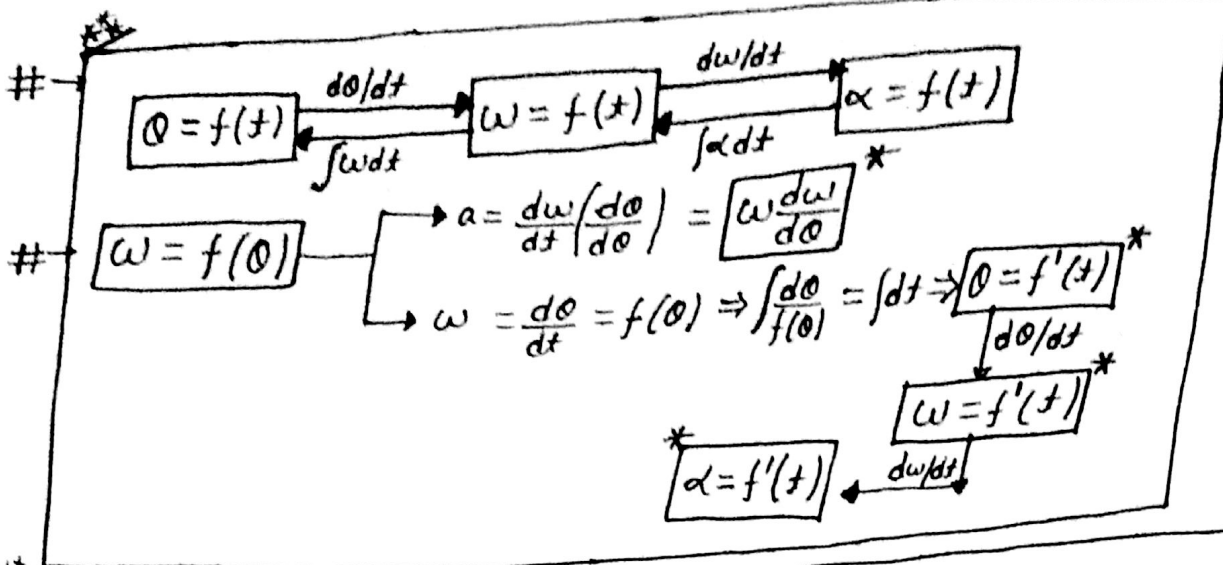
1a) -> Uniform Accn -> Magnitude + Direction -> same.

1b) -> Non-uniform Accn -> Magnitude/Direction/both change.

1c) -> Inst. Angular Accn -> $\alpha_t = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$

$\alpha_t = \frac{d\omega}{dt}$ = 1st derivative of Angular velo.

$\alpha_w = \frac{d}{dt} \left(\frac{d\theta}{dt} \right) = \frac{d^2\theta}{dt^2}$ = 2nd derivative of Angular disp.



Avg. Angular Acceleration

$$\vec{\alpha}_{Avg} = \frac{\vec{\omega}_f - \vec{\omega}_i}{t_f - t_i}$$

NOTE → Direction of Inst. Acceleration in the direction of torque but direction of Avg Angular Accn in the direction of change in the Angular velocity.

$\alpha = f(t) \rightarrow \langle \alpha \rangle = \alpha_{Avg} = \frac{\int_{t_1}^{t_2} f(t) dt}{\int_{t_1}^{t_2} dt}$

$\alpha = f(\theta) \rightarrow \langle \alpha \rangle = \frac{\int_{\theta_1}^{\theta_2} f(\theta) d\theta}{\int_{\theta_1}^{\theta_2} d\theta}$

Relation b/w Linear Accn & Angular Accn

$$v = \omega r$$

$$\frac{d(v)}{dt} = \frac{d(\omega r)}{dt}$$

$$\vec{a} = \vec{\omega} \times \vec{r}$$

Diagram: A circle of radius r rotating with angular velocity ω and angular acceleration α . A point on the circumference has linear velocity v and linear acceleration a . The vectors $\vec{r}$, $\vec{\omega}$, $\vec{a}$ are shown. The relationships $\vec{r} \perp \vec{\omega}$, $\vec{\omega} \perp \vec{v}$, and $\vec{\omega} \perp \vec{a}$ are indicated.

Eqn of Motion

Linear Motion

- * $v = u + at$
- * $s = ut + \frac{1}{2}at^2$
- * $v^2 = u^2 + 2as$
- * $s_{nth} = u + \frac{1}{2}a(2n-1)$

$\left[\begin{array}{l} u = \text{Initial velocity} \\ v = \text{Final velocity} \\ a = \text{Linear Acceleration} \\ s = \text{Linear displacement} \end{array} \right]$

NOTE → Eqn of Motion is used only when Angular Accn remain same.

$\left[\begin{array}{l} s \rightarrow \theta \\ u \rightarrow \omega_0 \\ v \rightarrow \omega \\ a \rightarrow \alpha \end{array} \right]$

Angular Motion

- * $\omega = \omega_0 + \alpha t$
- * $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
- * $\omega^2 = \omega_0^2 + 2\alpha\theta$
- * $\theta_n^{th} = \omega_0 + \frac{1}{2}\alpha(2n-1)$

$\left[\begin{array}{l} \omega_0 = \text{Initial Angular velocity} \\ \omega = \text{Final Angular velocity} \\ \alpha = \text{Angular Acceleration} \\ \theta = \text{Angular displacement} \end{array} \right]$

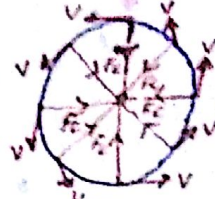
Centripetal force & Acceleration

When particle rotate in a circular path direction of particle continuously change that's why force require to change the direction that's why require force in circular motion is called centripetal force. Its direction is always towards centre of circular path.

$$F_c = \frac{mv^2}{r} = \frac{m(\omega r)^2}{r} = m\omega^2 r = m(\omega r)$$

$$\vec{F}_c = \frac{mv^2}{r} (-\hat{r}) = m\omega^2 r (-\hat{r}) = m(\vec{\omega} \times \vec{v}) (-\hat{r})$$

$\hat{r}$ = unit vector along the Radial vector.
 v = Linear (tangential) velocity [Tangential vector]
 ω = Angular velocity [Axial vector]



Centripetal Accn ($\vec{a}_c$)

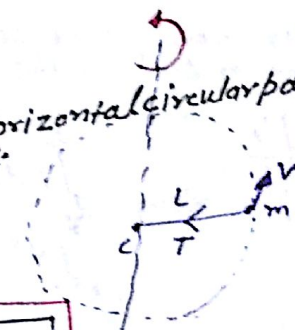
$$\vec{a}_c = \frac{\vec{F}_c}{m} = \frac{v^2}{r} (-\hat{r}) = \omega^2 r (-\hat{r}) = \vec{\omega} \times \vec{v} (-\hat{r})$$

NOTE → * centripetal force is necessary for circular motion but it can't change speed of particle.

* centripetal force is provided by natural forces like gravitational, Electrostatic, Friction force etc.

** Ex → of centripetal force

ii) → When particle attach with string & rotate in horizontal circular path & tension of string is provided by necessary centripetal force.



$$F_c = T$$

$$T = \frac{mv^2}{L} = m\omega^2 L = m(\omega L)$$

$$T_{max} = \frac{mv_{max}^2}{L} = m\omega_{max}^2 L$$

$$v_{max} = \sqrt{\frac{T_{max} L}{m}}$$

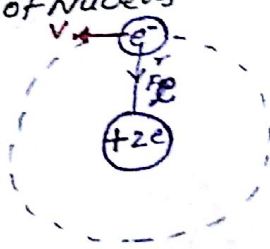
$$\omega_{max} = \frac{T_{max}}{mL}$$

T_{max} = Max possible tension in string

iii) → Electron rotate around the nucleus then necessary centripetal force is provided by Electrostatic attractive force of nucleus.

$$F_c = F_e = \frac{k(ze)(e)}{r^2}$$

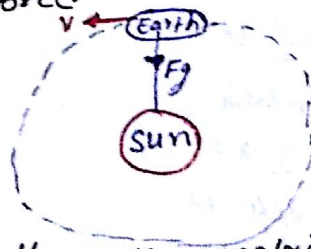
$$\frac{mv^2}{r} = \frac{k(ze^2)}{r^2}$$



iiii) → When Earth rotate around the sun then necessary centripetal force is provided by gravitational attractive force.

$$F_c = F_g = \frac{G(M_s)(M_e)}{r^2}$$

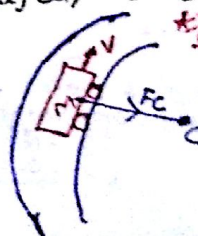
$$\frac{m_e v_e^2}{r} = \frac{G(M_s)(M_e)}{r^2}$$



iv) → In horizontal circular track to turn safely necessary centripetal force is provided by friction force.

$$\mu(mg) \geq \frac{mv^2}{r}$$

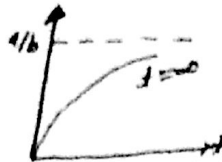
$$v_{safe} = \sqrt{\mu r g} \propto m^0$$



****NOTE** → Max safe velo depend on friction coefficient, Radius of circular path, Gravitation Accn & It is independent from Mass of vehicle.

Angular Accn of particle is change with Angular velocity as Relation $\alpha = a - bw$.
Where a & b is const. & particle start from Rest. then Angular velocity of particle at time t .

Previous script of
 $\alpha = \frac{dw}{dt} = a - bw$
 Any $\left[w = \frac{a}{b} (1 - e^{-bt}) \right]$



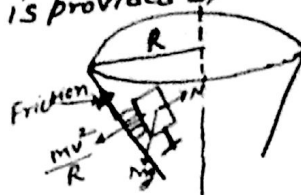
$t \uparrow \Rightarrow e^{bt} \uparrow + e^{-bt} = \frac{1}{e^{bt}} \rightarrow (1 - e^{-bt}) \uparrow$

~~Centrifugal force~~ Acceleration

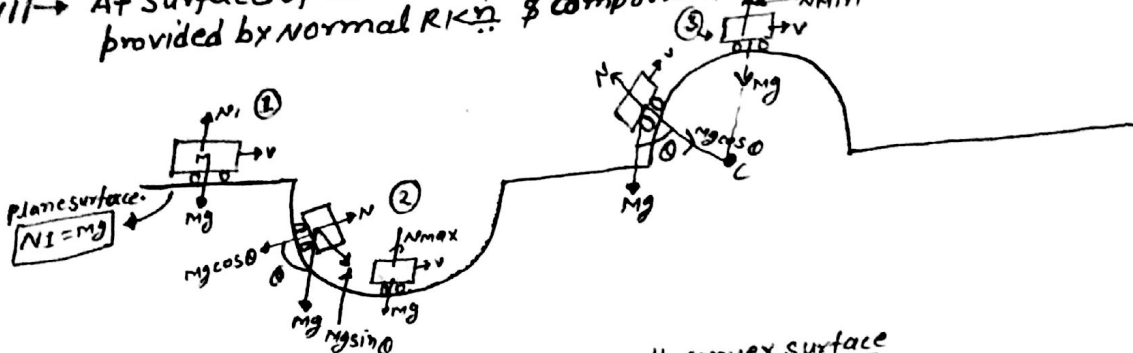
IV) → In a death Well necessary centripetal force is provided by Normal Reaction.

$V \geq \sqrt{\frac{Rg}{11}}$

$V_{\text{minimum safe velo}} = \sqrt{Rg/11} \propto m^0$



Vii) → At surface of concave & convex bridge necessary centripetal force is provided by Normal $R < N$ & component of weight.



Concave surface

$N - Mg \cos \theta = F_c = \frac{Mv^2}{r}$

$N = Mg \cos \theta + \frac{Mv^2}{r}$

$(N_{\text{max}})_2 = Mg + \frac{Mv^2}{r}$

Bottom point

convex surface

$Mg \cos \theta - N = F_c = \frac{Mv^2}{r}$

$N = Mg \cos \theta - \frac{Mv^2}{r}$

$N_{\text{min}} = Mg - \frac{Mv^2}{r}$

Top point

$V \uparrow \Rightarrow N_1 = \text{same}$ | # $N_3 \geq 0 \Rightarrow \text{Safe condn.}$
 $N_2 \uparrow$ | $N_3 < 0 \Rightarrow \text{Unsafe condn.}$
 $N_3 \downarrow$

Max possible velocity at top point for safe condition

$N_3 \geq 0$

$Mg - \frac{Mv^2}{r} \geq 0$

$\frac{Mv^2}{r} \leq Mg \Rightarrow v \leq \sqrt{rg}$

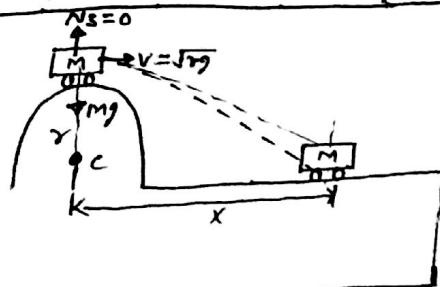
$V_{\text{max}} = \sqrt{rg}$

For Speed breaker

$X = u \sqrt{\frac{2h}{g}}$

$= (\sqrt{rg}) \sqrt{\frac{2r}{g}}$

$X = \sqrt{2r} = 1.414r$



VIII → By rotation of container, all liquid particles moves in circular path & necessary 'centripetal force is provided by pressure difference along radial direction', hence there is a pressure difference in horizontal direction due to this rotation.

$$\frac{dp}{dr} = \rho r \omega^2$$



Centrifugal force (Pseudoforce)

When particle moves in a circular path then from non-inertial frame of reference, force is appeared outward from centre of circular path which is called pseudoforce. & Its direction radially outward.

$$F = ma \quad * \quad F = \frac{mv^2}{r} (\hat{r}) = m\omega^2 r (\hat{r}) = m(\omega v) \hat{r}$$

NOTE → Magnitude of centrifugal force is equal to centripetal force.

Types of circular motion

1) → Uniform circular motion

$$* |\vec{v}_1| = |\vec{v}_2| = |\vec{v}_3| = |\vec{v}_4| \Rightarrow \text{same}$$

$$* \vec{\omega} = \text{same}$$

$$* \vec{a} = 0, \vec{a}_t = 0$$

$$* |\vec{a}_c| = \frac{v^2}{r}$$

$$* a_{\text{net}} = \vec{a}_c = \frac{v^2}{r} (-\hat{r})$$

$$* F_{\text{net}} = F_c = \frac{mv^2}{r} (-\hat{r})$$

$$* \vec{v}, \vec{p} \Rightarrow \text{change (magnitude same, direction change)}$$

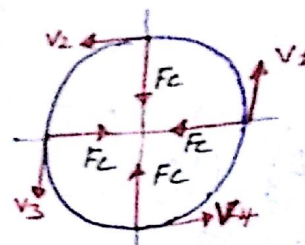
$$* \text{change in velocity } |\Delta \vec{v}| = 2v \sin(\theta/2)$$

$$* \text{change in Linear Momentum}$$

$$\Delta \vec{p} = 2mv \sin(\theta/2)$$

$$= 2p \sin(\theta/2)$$

$$* \text{Avg. velocity } |\vec{v}_{\text{avg}}| = v \frac{\sin(\theta/2)}{(\theta/2)}$$



$$* \text{Avg. Acceleration}$$

$$|\vec{a}_{\text{avg}}| = \frac{v^2}{r} = \frac{\sin(\theta/2)}{(\theta/2)}$$

$$* K \cdot E = \text{some} \Rightarrow \Delta K \cdot E = 0$$

$$* \text{Work done by centripetal force}$$

$$(\vec{F} \perp \vec{v}) \parallel \vec{s}$$

$$* P = \vec{F}_c \cdot \vec{v} = F_c v \cos 90^\circ = 0$$

2) → Non-uniform circular motion

$$* |\vec{v}_1| \neq |\vec{v}_2| \neq |\vec{v}_3| \neq |\vec{v}_4| \Rightarrow \text{Speed change}$$

$$* \vec{\omega} = \text{change}$$

$$* \vec{a} \neq 0, \vec{a}_t = \frac{dv}{dt} \neq 0$$

$$* |\vec{a}_c| = \frac{v^2}{r}, F_c = \frac{mv^2}{r} \neq 0$$

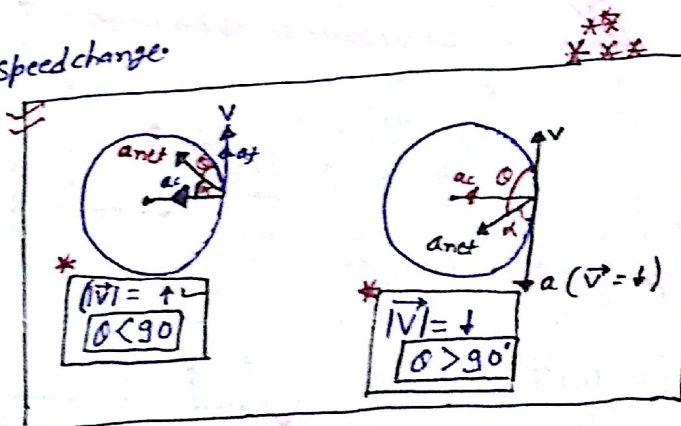
$$* |\vec{a}_{\text{net}}| = \sqrt{a_t^2 + a_c^2}$$

$$* \text{angle of } |\vec{a}_{\text{net}}| \text{ from } |\vec{a}_c|$$

$$\alpha = \tan^{-1} \left(\frac{a_t}{a_c} \right)$$

$$* |\vec{F}_{\text{net}}| = \sqrt{F_c^2 + F_t^2}$$

$$* \vec{v}, \vec{p}, K \cdot E = \text{change}$$



$$* W = \vec{F}_t \cdot d\vec{s} + \vec{F}_c \cdot d\vec{s} \neq 0$$

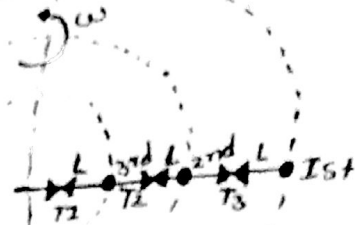
$$* P = \vec{F}_c \cdot \vec{v} + \vec{F}_t \cdot \vec{v} \neq 0$$

3-particle of mass 'm' is attached with string as shown & rotate in a horizontal circular path with uniform angular velocity. then find ratio of tension in different path of string.

- 1st $\rightarrow T_3 = m\omega^2(2L)$ (i)
 2nd $\rightarrow T_2 = T_3 = m\omega^2(2L)$ (ii)
 3rd $\rightarrow T_2 - T_1 = m\omega^2(L)$ (iii)

After finding
 $T_2 = 5m\omega^2 L$
 $T_1 = 6m\omega^2 L$

$T_1 : T_2 : T_3 = 6m\omega^2 L : 5m\omega^2 L : 3m\omega^2 L$
 $= 6 : 5 : 3$



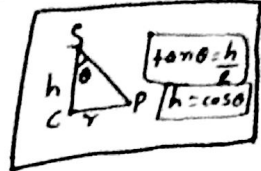
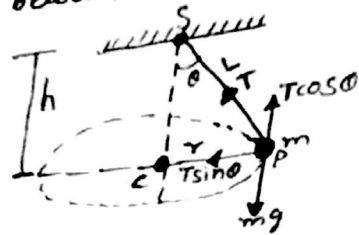
Particle of mass 'm' attached with string of length 'L' & rotate in a horizontal circular path of radius 'r'. If centre of circular path 'h' below the point of suspension then time period of oscillation.

$\tan \theta = \frac{\omega^2 r}{g}$

$\omega = \sqrt{\frac{g \tan \theta}{r}}$

$\omega = \sqrt{\frac{g}{h \cos \theta}}$

$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{h}{g \cos \theta}} = 2\pi \sqrt{\frac{L \cos \theta}{g}}$



Particle of mass 'm' rotate in circular path of radius 'r' in a bowl of radius 'R'. If centre of circular path 'h' height below from centre of bowl then time period of oscillation.

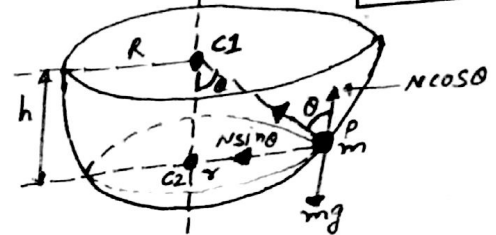
$\tan \theta = \frac{\omega^2 r}{g}$

$\omega = \sqrt{g \tan \theta}$

$\omega = \sqrt{\frac{g}{r} \tan \theta}$

$\omega = \sqrt{\frac{g}{h \cos \theta}}$

$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{h}{g \cos \theta}} = 2\pi \sqrt{\frac{R \cos \theta}{g}}$



$\tan \theta = \frac{r}{h}$

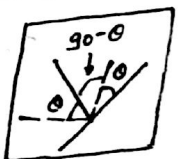
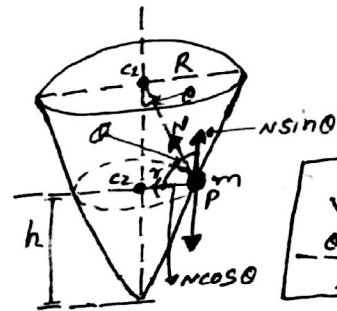
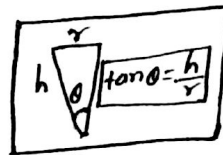
Same as previous Q.

$\tan \theta = \frac{g}{\omega^2 r}$

$\omega = \sqrt{\frac{g}{r \tan \theta}}$

$\omega = \sqrt{\frac{g h}{r^2}}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{g h}} r$

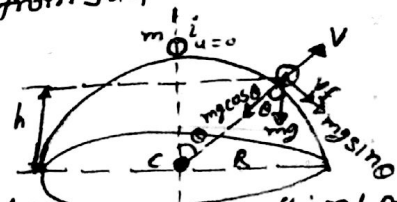


Particle of mass 'm' drop from top point having radius 'R'. Surface is frictionless. then height from the bottom where it leave the contact from surface.

$N = mg \cos \theta = \frac{mv^2}{R}$ (i)

$\cos \theta = \frac{2(R-h)}{R}$ (ii)

$h = \frac{2R}{3}$



Particle of mass 'm' placed at distance 'r' from centre of disc, friction coefficient of surface is 'u' then max. angular velocity of disc so that particle not at rest.

$F_f \geq m\omega^2 r$

$\mu mg \geq m\omega^2 r$

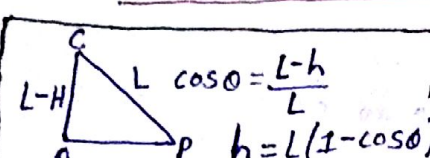
$\omega \leq \sqrt{\frac{\mu g}{r}}$

$\omega_{\max} = \sqrt{\frac{\mu g}{r}}$



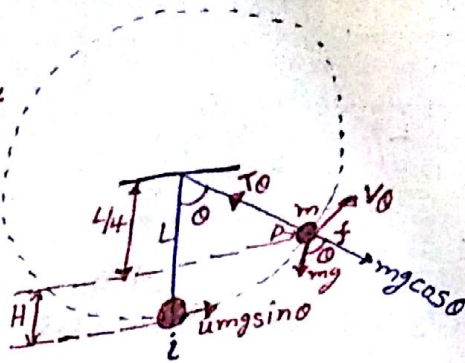
Vertical circular motion

ii) → Velocity at 'O' angular position



$$VO = \sqrt{u^2 - 2gh}$$

$$= u^2 - 2gL(1 - \cos\theta)$$



iii) → Tension at 'O' angular position

$$T_0 = mg \cos\theta + \frac{mv_0^2}{L}$$

$$T_0 = mg \cos\theta + \frac{m}{L} [u^2 - 2gL(1 - \cos\theta)]$$

$$T_0 = \frac{mu^2}{L} + mg(3 \cos\theta - 2)$$

* u = velocity at bottom.
 * θ = Angular position
 * L = Length of string / Radius of circular path.

iiii) → At bottom point ($\theta = 0$)

$$V_{\text{bottom}} = u$$

$$T_{\text{bottom}} = \frac{mu^2}{L} + mg$$

iv) → At Horizontal point ($\theta = 90^\circ$)

$$V_{\text{Horizontal}} = \sqrt{u^2 - 2gL}$$

$$T_{\text{Horizontal}} = \frac{mu^2}{L} - 2mg \Rightarrow \frac{mu_H^2}{L}$$

v) → Top point ($\theta = 180^\circ$)

$$V_{\text{Top}} = \sqrt{u^2 - 4gL}$$

$$T_{\text{Top}} = \frac{mu^2}{L} - 5mg$$

* $T_{\text{max}} - T_{\text{min}} = T_B - T_T = 6mg$
 * $T_B - T_H = 3mg$
 * $T_H - T_T = 3mg$

$$V_{\text{bottom}} = u$$

$$V_{\text{Horizontal}} = \sqrt{u^2 - 2gL}$$

$$V_{\text{Top}} = \sqrt{u^2 - 4gL}$$

$$T_{\text{bottom}} = \frac{mu^2}{L} + mg = T_{\text{max}}$$

$$T_H = \frac{mu^2}{L} - 2mg$$

$$T_{\text{Top}} = \frac{mu^2}{L} - 5mg = T_{\text{min}}$$

Condition to complete vertical circular path.

$$T_{\text{min}} = T_{\text{Top}} \geq 0$$

$$\frac{mu^2}{L} - 5mg \geq 0$$

$$u \geq \sqrt{5gL}$$

* $(u_{\text{bottom}})_{\text{min}} = \sqrt{5gL}$

* $T_{\text{bottom}} = 6mg$

* $V_H = \sqrt{3Lg}$

* $V_T = \sqrt{gL}$

* $T_{\text{Top}} = 0$

$$T_B \geq 6mg$$

$$T_H \geq 3mg$$

$$T_T = 0$$

$$V_B \geq \sqrt{5Lg}$$

$$V_H \geq \sqrt{3Lg}$$

$$V_T \geq \sqrt{Lg} \Rightarrow V_{\text{min at top point}} \rightarrow \text{Critical velocity.}$$

AI PMT 2026

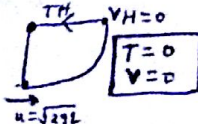
i) → $\sqrt{2Lg} < V_{\text{bottom}} < \sqrt{5Lg}$

* $L < h < 2L$, $T = 0$
 * $90^\circ < \theta < 180^\circ$, $V \neq 0$



ii) → $V_{\text{bottom}} = \sqrt{2Lg}$

* $L = h$, $\theta = 90^\circ$



iii) → $0 < u < \sqrt{2Lg} \Rightarrow$ simple pendulum

* $h < L$
 * $\theta < 90^\circ$
 * $T \neq 0$
 * $V \neq 0$

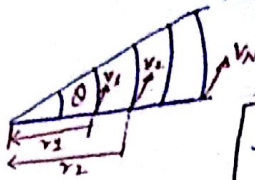
$T_{\text{bottom}} > T_{\text{Horizontal}} > T_{\text{top}}$
 $T_{\text{bottom}} - T_{\text{top}} = 6mg$
 $T_{\text{bottom}} - T_{\text{Horizontal}} = 3mg$
 $T_{\text{Horizontal}} - T_{\text{top}} = 3mg$

$u_{\text{bottom}} = \sqrt{4g} \Rightarrow \text{parabolic}$
 $u_{\text{bottom}} = \sqrt{2Lg} \Rightarrow \theta = 90^\circ \Rightarrow \text{Horizontal}$
 $u_{\text{bottom}} = \sqrt{Lg} \Rightarrow \theta < 90^\circ \Rightarrow \text{not horizontal or, simple pendulum.}$

Magnitude of change in velocity b/w Horizontal & bottom $\rightarrow \sqrt{2(u^2 - gL)}$

When two or more than two particles rotate in horizontal circular path -

$\frac{\omega}{r} = \omega = \text{same} = \frac{v}{r}$
 $\frac{v_1}{r_1} = \frac{v_2}{r_2} = \dots = \frac{v_n}{r_n}$



Vechle of mass 'M' is drop from height 'H' as shown. than value of 'R' if it will complete $\frac{1}{2}$ circular motion as shown in fig.

$$v = \sqrt{2g(H-h)} \geq \sqrt{5Rg}$$

$$2g(H-h) \geq 5Rg$$

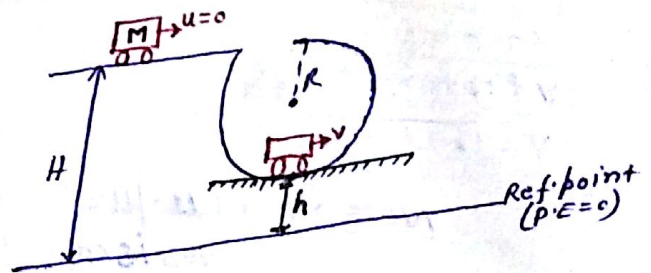
$$R \leq \frac{2}{5}(H-h)$$

$$H-h \geq \frac{5}{2}R$$

$$H \geq \frac{5}{2}R + h$$

$$h=0 \Rightarrow H \geq \frac{5}{2}R$$

$$R \leq \frac{2}{5}H$$

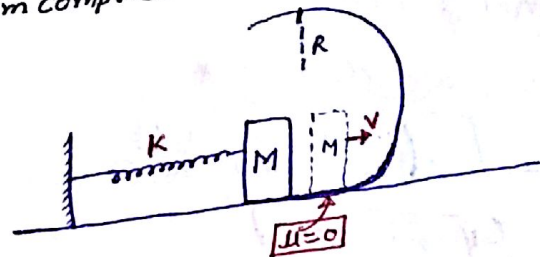


Mass 'M' is compressed with spring of spring coefficient 'K' & Release. If it will complete vertical circular path as shown. then minimum compression in spring.

$$v = x\sqrt{\frac{K}{m}} \geq \sqrt{5Rg}$$

$$x_{\min} \geq \sqrt{\frac{m}{K}(5Rg)}$$

$$x_{\min} = \sqrt{\frac{m}{K}(5Rg)}$$



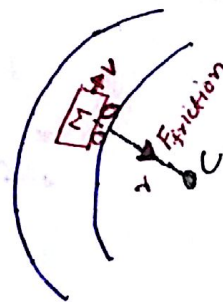
Circular Motion in Daily Life

11) Skidding condition

$$\mu mg \geq \frac{mv^2}{r}$$

$$v \leq \sqrt{\mu rg}$$

$$v_{\text{maximum safe velo.}} = \sqrt{\mu rg}$$



$r = \text{Radius}$
 $\mu = \text{Friction b/w Road \& Tyre for safe driving without skidding.}$

2016 AIEEE

12) overturning condition (toppling condition)

$$R_1 + R_2 = Mg \quad \text{--- (i)}$$

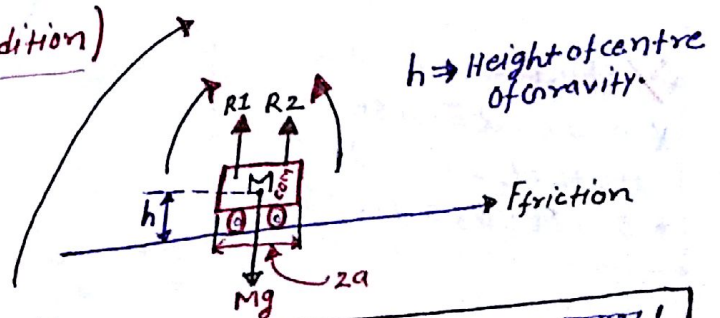
$$R_1 - R_2 = \left(\frac{mv^2}{r}\right) \frac{h}{a} \quad \text{--- (ii)}$$

Normal Reaction on outer Wheel.

$$R_1 = \frac{Mg}{2} \left[1 + \left(\frac{v^2}{rg} \right) \left(\frac{h}{a} \right) \right]$$

Normal Reaction on Inner Wheel.

$$R_2 = \frac{Mg}{2} \left[1 - \left(\frac{v^2}{rg} \right) \left(\frac{h}{a} \right) \right]$$



* वही vechle पलटने का chance देता है!
 * जितना down होगा C.M पलटने का chance कम!

Safe condition

$$* V \uparrow, R \uparrow, h \downarrow$$

$R \downarrow \Rightarrow$ Normal Rsp on inner wheel.

$$V \leq \sqrt{rg \left(\frac{a}{h} \right)}$$

$$V_{\text{max Safe velocity}} = \sqrt{rg \left(\frac{a}{h} \right)}$$

NOTE $\rightarrow$ To avoid skidding & overturning in plane, circular path safe velocity of vehicle is less than $\sqrt{ug}$ & $\sqrt{rg \left(\frac{a}{h} \right)}$

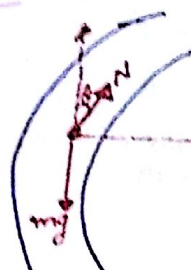
Bending of cyclist in a circular path

$$N \sin \theta = \frac{mv^2}{r}$$

$$\tan \theta \propto v^2$$

$$V \uparrow \Rightarrow \tan \theta \uparrow$$

$$V = \sqrt{rg \tan \theta}$$



* $r \Rightarrow$ Radius of circular path.
* $\theta \Rightarrow$ Bending angle from vertical.

NOTE $\rightarrow$ * If $u = \sqrt{rg \tan \theta}$, then frictional force on vehicle is zero.
this is called ideal speed.

* If $u < \sqrt{rg \tan \theta}$, then fr. force will be outward.

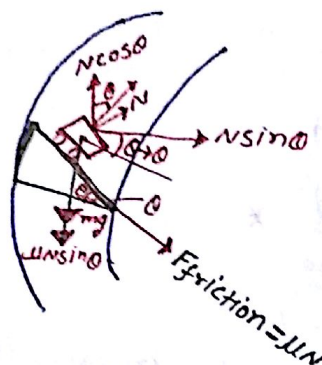
Banking of Track

Max safe velocity

$$V = \sqrt{rg \left(\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \right)}$$

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(If fr. $\oplus$ int.)



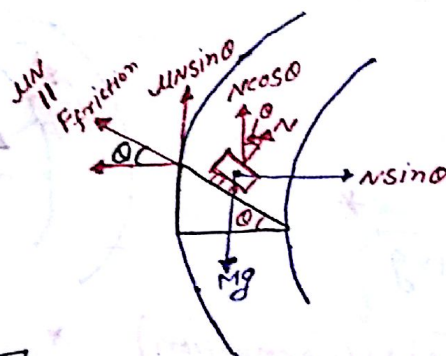
$$\mu = 0$$

$$V = \sqrt{rg \tan \theta}$$

$$\tan \theta = \frac{h}{w}$$

Minimum safe velocity

$$V = \sqrt{rg \left(\frac{\tan \theta - \mu}{1 + \mu \tan \theta} \right)}$$



Standard

$$* \omega_0 = 0, \alpha = \text{const.}$$

$$t_1 = t_2 = t_3 = \dots = t_n$$

$$* \theta_1 : \theta_2 : \theta_3 = 1 : 3 : 5 : 7 \dots$$